RANDOM WALK AND PRICE TRENDS: THE LIVE CATTLE FUTURES MARKET

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A NOTABLE and provocative development in the recent literature has been the application of the theory of random walks to the analysis of price behavior in the stock and commodity futures markets. The basic hypothesis of the random walk theory is that a particular price series behaves as a simple stochastic process. Successive price changes are independent random variables, implying that the past history of a series generates no information that would be useful in predicting future price changes.

Several authors have tested this independence hypothesis, but with mixed results. Many of the studies on stock market prices are contained in a book edited by Cootner [5], and some show evidence that stock prices behave as a random walk,[1] while others reject the theory.[2] Similar recent testing on commodity futures prices has added to the controversy. Tied to this assortment of results has been a variety of analytical techniques, each with its own justification, ranging from sophisticated statistical tools, such as spectral analysis, to mechanical trading rules which attempt to generate speculative profits.

Among the studies on commodity futures prices, Larson [13], using autocorrelograms, found evidence to support Working's [17] theory of anticipatory prices which implies that prices move randomly. Stevenson and Bear [16], who used an assortment of statistical tools and mechanical trading rules, concluded that corn and soybean futures prices move in a systematic rather than a random fashion. This tended to agree with earlier work done by Houthakker [11] who applied a stop-loss scheme to corn trading. On the other hand, Cargill and Rausser [3], utilizing spectral analysis on various futures contracts for 1967, including corn, concluded that "a simple stochastic process appears consistent with commodity markets price behavior."

A major shortcoming of the entire analysis and a possible reason for the lack of wide acceptance for any single result is the failure to subject identical data to both statistical and mechanical filter tests. The consequence has been varying results, depending upon data and tests used. Only Stevenson and Bear have attempted to apply alternative statistical tools and mechanical trading rules to similar data. However, as will be noted later, one may quarrel with their choice of statistical tools—namely, serial correlation and runs analysis. It is the purpose of this paper to rectify this void in the literature by applying both a sophisticated statistical tool and mechanical filter tests to similar data. Spectral analysis along with several mechanical filters will be employed to test whether the live cattle futures prices move in a random or a systematic fashion.

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1. [5, pp. 85, 139, 162]. Also, see [6], [7], [8], [18] for similar results.
2. [5, pp. 231, 253, 338].

I. Model, Data, and Tests Used

Model—If the random walk model is in fact true, then:

$$X_t - X_{t-1} = \varepsilon_t \qquad (1)$$

where X_t is the discrete price series the model suggested and ε_t has mean zero and is uncorrelated with ε_{t-k}, all $k \neq 0$. The ε_t series is called white noise. Thus, if the model is correct, the series will proceed by a sequence of unconnected steps, starting each time from the previous value of the series.

Data—The data employed to investigate the random walk model are daily closing prices of 30 live beef cattle futures contracts. They start with the first contract traded,[3] April 1965, and include the subsequent June, August, October, December, and February contracts for each year, up through and including February 1970. This amounts to 6,914 observations, or an average of about 230 observations per contract. Each series is terminated the next to the last day of trading because delivery often occurs on the last trading day, creating an extreme price change.[4] If the closing price on any day was a range, the midpoint of the range was selected as the observation.[5]

Tests—Several statistical tools exist and have been used for testing the appropriateness of the random walk model as a representative of stock or commodity market prices. Among these are serial correlation tests and analysis of runs of successive price changes of the same sign. Fama [6] used these techniques on the stock market and found that they upheld the random walk model.[6] On the other hand, Stevenson and Bear [16] rejected the model after using the techniques on commodity prices.

There has been some criticism of the serial correlation and runs analysis tests, particularly by Fama himself [6]. Primarily, these approaches are too unsophisticated or too restrictive to pick up complicated patterns of price movements. Because of these weaknesses, this study employs spectral analysis as the technique for testing for independence. Spectral analysis is a powerful statistical tool which indicates whether or not a time series is random by establishing confidence intervals around its estimates.

II. Spectral Analysis

Spectral analysis has been used by Granger and Morgenstern [10] and Godfrey, Granger and Morgenstern [8] to test the random walk hypothesis against stock market price indices, while Cargill and Rausser [3] used the technique on selected commodity market price series. Each study found evidence to support the contention that prices behave in a random walk manner.

3. Trading on the first four contracts began November 30, 1964.

4. An anonymous reviewer pointed out that delivery conditions may alter the price-generating mechanism during the last few days of each series. While this may be true, a check of the data and results indicates that an earlier termination of each series would not change the basic conclusions drawn in this study.

5. The data source was [4].

6. Also see [5, pp. 85, 139].

Theory—The spectral method decomposes a time series into a number of components, each associated with a frequency or period.[7] Frequency indicates the number of cycles per unit of time, and the period describes the length of time required for one complete cycle. This spectral decomposition of a time series yields a spectral density function and measures the relative importance of each of the frequency bands in terms of its contribution to the overall variance of the time series. Essentially, spectral analysis is an examination of the variance of a time series with respect to frequency components [14].

The power density function is based on a Fourier transformation of the autocovariance of a stationary series (X_t, $t = 1, \ldots, n$), approximated by:

$$f(\omega) = \frac{1}{2\pi} \sum_{t=-\infty}^{\infty} \gamma(t) \cos \omega t \tag{2}$$

where $\gamma(t)$ denotes the autocovariance function.

Before the transformation from a time domain to a frequency domain is made, the autocovariance function is weighted with the Parzen lag window. The weights are of form [12, p. 244; 9, p. 61]:

$$W(k) = \begin{cases} 1 - \dfrac{6k^2}{m^2}\left(1 - \dfrac{k}{m}\right), \\ 0 \leqslant k \leqslant \dfrac{m}{2} \\ 2\left(1 - \dfrac{k}{m}\right)^3, \\ \dfrac{m}{2} < k \leqslant m \end{cases}$$

where m is the number of lags and k is the time-span between terms in the autocovariance function.

These weights are used so that statistically consistent estimates of the power density function can be obtained. Rather than estimate the power associated with a precise frequency, this technique estimates the average power centered around the frequency in question. This is statistically equivalent to averaging over the periodogram. The Parzen weights have the advantage over alternative weighting schemes by (1) allowing for a smaller leakage between frequency bands, (2) giving only positive estimates of the spectrum, and (3) allowing for a larger number of degrees of freedom [14, pp. 111-112].

The spectral estimates are random variables for which tests of significance have been developed. Before the tests for relative peaks are made, the spectrum

7. For the basic theory and estimation procedures underlying spectral analysis see Granger and Hatanaka [9] and Jenkins and Watts [12]. For model development and application similar to the method used in this study, see Cargill and Rausser [3] and Rausser and Cargill [14]. This study differs from the previous two primarily by the data used and subsequent results.

is normalized by dividing it by the variance of the series [12, p. 235]. This gives a theoretical value of white noise equal to 1.0 for a normally distributed independent random series. The test of significance for a relative peak follows from a confidence band for normally distributed independent random variables and can be expressed as:

$$\Pr\left[\chi^2_{\alpha}(v) \leqslant \frac{v\hat{f}(\omega)}{f(\omega)} \leqslant \chi^2_{1-\alpha}(v)\right] = 1 - 2\alpha \quad (3)$$

where $\chi^2(v)$ are the standard chi-squared values, $v = 3.71n/m$ is the equivalent degrees of freedom, $\hat{f}(\omega)$ is the estimated spectrum of the series, and $f(\omega) = 1.0$. A relative peak outside a confidence interval, for example, 95 per cent, is said to be significantly different from white noise at the 95 per cent level of confidence. The peak represents an important frequency component within the time series.

Results—The daily price series were transformed into first differences according to expression (1).[8] This reduces the influence of possible trends and allows for a direct test of the random walk model. If the price series is generated by the random walk model, then the spectrum will approximate the spectrum of white noise, within a certain confidence limit. The spectra for each series were estimated at 50 frequency components, unless this number violated the rule of thumb that m should not exceed one-quarter of n. When this problem occurred, the spectra were estimated at fewer frequency components, depending upon the size of n.

Two of the spectra are shown for illustrative purposes in Figures 1 and 2.

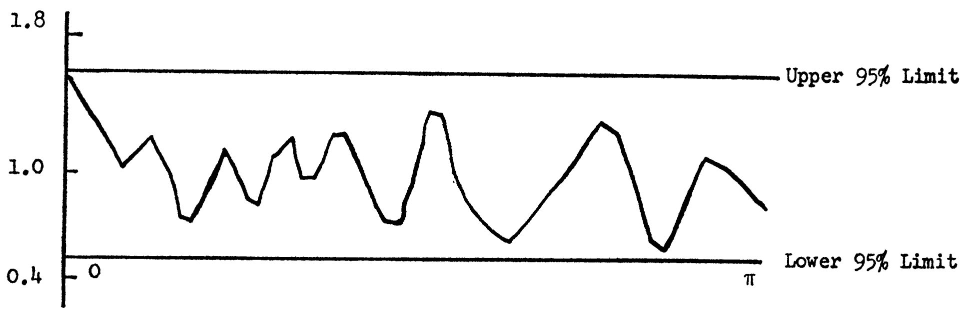

FIGURE 1
February 1968 Contract

The random walk model fits the data very well for the February 1968 contract, but the February 1970 contract indicates that a cyclical pattern exists in the data. A summary of the results of all 30 contracts is given in Table 1. Each contract was judged either random, almost random, or not random. The indefinite category of almost random was established for those contracts where a few estimated points fell slightly outside the confidence band. Since the bands were drawn at the 95 per cent level, these few points may well be within

8. Data were also transformed into first differences of logarithms, but subsequent analysis yielded results very similar to those reported.

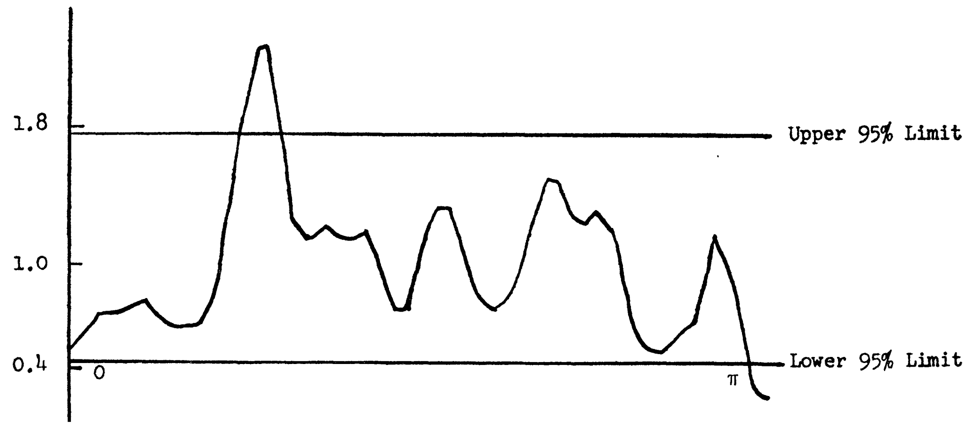

FIGURE 2
February 1970 Contract

99 per cent confidence bands. Also, spectral results have to be interpreted cautiously because $v\hat{f}(\omega)/f(\omega)$ is only approximately distributed as $\chi^2(v)$ and spectral estimates are sensitive to extreme values. Thus, unless all the points were within the bands (judged random), or a definite peak existed as in Figure 2 (judged not random), the contract was put into the almost random class. More detailed results with this classification are given later in this paper.

It is coincidental that the last 6 contracts traded were all judged not random. This occurred because each was in existence during the abnormal time period in 1969 when cattle prices rose dramatically from $29.50/cwt. to over $34.00/cwt., then fell back to $29.00/cwt., all in about six months. The prices in the remaining 24 contracts are either random (13 contracts) or almost random (11 contracts), and these contracts are intermingled.

Thus, this analysis indicates that a simple stochastic process appears consistent with live beef cattle futures price behavior part of the time, but not at other times. Presumably then, drawing upon the conclusion in previous studies [3, 15], if a price series is random, it is not possible to extrapolate from these price movements a mechanism which can be used to generate profitable invest-

TABLE 1
RESULTS OF SPECTRAL ANALYSIS ON 30 LIVE CATTLE FUTURES CONTRACTS

Contract Month	Total Number of Years	Number of Years: Random	Almost Random	Not Random
February	5	3	1	1
April	5	2	2	1
June	5	1	3	1
August	5	2	2	1
October	5	3	1	1
December	5	2	2	1
		13	11	6

ment rules. Conversely, if prices are not a random walk, profitable mechanical rules for investment may be possible. It is to these arguments that further comparative analysis is needed.

III. Filter Rules

Mechanical trading rules, called filters, are simulation models which test for the possibility of nonlinear dependency existing in the price data. Most statistical tools, including spectral analysis, are incapable of detecting such relationships. Many professional traders, chartists especially, claim that nonlinear patterns and dependence do exist in price data, allowing them to generate speculative profits, despite results based on the use of statistical tools indicating randomness.

Model—S. A. Alexander [1] was the first to apply mechanical filter rules to security prices; Fama and Blume [7] have added to the literature. Houthakker [11], and more recently, Stevenson and Bear [16] have applied various filters to grain commodity prices. The rules employed in this study are defined as follows: If the daily closing price of a cattle contract moves up at least x per cent, buy and hold the contract until its price moves down at least x per cent from a subsequent high, at which time simultaneously sell and go short. The short position is maintained until the daily closing price rises at least x per cent above a subsequent low, at which time one reverses position and goes long.[9]

The application of mechanical filter rules on commodity futures prices circumvents some problems that were encountered in previous studies on the stock market. For instance, (1) the rules are applied directly to prices not indices; and (2) dividends are not declared on commodities as they are on stocks, thereby eliminating possible sources of bias. The problem of determining the statistical significance of the results does remain for any price series, however. It is not possible to determine the likelihood of similar results if the process indeed has independent increments; therefore, probability statements cannot be made.

Procedure—The filter technique as defined above is applied to the same data as was the spectral analysis, the daily closing prices of 30 live cattle futures contracts. The filter rules used in this study are 1, 2, 3, 4, 5, and 10 per cent.[10] All transactions were made at the closing price which triggered the decision, and not at a price exactly x per cent from the initial price.[11] The market position held on the next to the last day of trading was liquidated at that closing price, regardless of the position or whether the price had not yet moved x per cent since the preceding transaction.

Contracts for live beef cattle traded on the Chicago Mercantile Exchange from November 1964 through February 1970 were generally for 25,000

9. Other mechanical filters exist, but none have proved very successful [16].

10. Prices do not often fluctuate much more than 10 per cent during the duration of a contract.

11. Buying and selling at a price exactly x per cent from an initial price introduced severe biases and problems for Alexander [2].

pounds with a commission of $36 per round-trip. Thus, profits or losses for each round-trip were based on these data and on the actual prices traded.[12] Also, the typical margin requirement requested by the broker was $300.[13] Thus, a return on investment, based on this margin and net returns, was computed for each contract-filter rule, then adjusted to an annual basis for comparisons.

Results—Table 2 indicates for each filter rule, totaled over the 30 contracts, the gross return, number of round-trips, net return after commissions, and annual net rate of return based on the $300 investment. The total gross return for each filter is positive, but the commissions for the 1 and 5 per cent rules caused their net returns to be negative. The 3 per cent rule has the largest net return, followed by the 2, 10, and 4 per cent rules. As would be expected, the smaller the filter rule, the more often the contract is traded.

TABLE 2
RESULTS OF FILTER RULES

Filter Rule	Gross Return	Number of Round-trips	Net Return	Annual Net Rate of Return (per cent)
1 per cent	+$17,390.00	657	−$6,262.00	− 76.75
2 per cent	+ 14,045.00	271	+ 4,289.00	+ 54.78
3 per cent	+ 14,515.00	161	+ 8,719.00	+115.82
4 per cent	+ 5,025.00	126	+ 489.00	+ 7.08
5 per cent	+ 872.50	101	− 2,763.50	− 44.37
10 per cent	+ 2,917.50	34	+ 1,693.50	+ 45.30

It was possible to generate gross profits much larger than those shown in Table 2 by using 5, 10, and 15 day moving averages. However, this procedure led to more frequent trading, so that after commissions the net profits indicated the scheme was far inferior to the percentage rules. Consequently, these results are not included.

The high gross return for the small filter rules indicates that trading on the basis of mechanical decisions rules can be very profitable, providing one does not have to pay commission fees. Of course, floor traders do not pay the $36 commission, only a small maintenance charge. Thus, to them prices do fluctuate so that considerable profits are available. Furthermore, profits even seem possible after commissions for the average trader for four of the six filters. But do these results deny the random walk hypothesis?

12. Contract size increased to 40,000 pounds effective with the August 1969 contract. However, for comparative purposes profits and losses were based on 25,000 pounds for all contracts. The effect of the larger contracts would only be to increase the amount of profit or loss per trade.

13. It is recognized that this margin may differ among brokers and over time. However, in order to determine the investment effects of just the price series itself, it is necessary to hold variables such as contract size, commission, and margin requirement constant. Little realism of actual market trading is lost by such action. Also, brokers sometimes require additional margin for their protection if the price moves in the wrong direction, but this is ignored for the same reasons as above and because investment returns are not critical to this study.

IV. Comparison of Results

Alexander [1], Fama and Blume [7] and Stevenson and Bear [16] compared their filter rules with a buy-and-hold policy, concluding that if the filter rule was more profitable than buy-and-hold, prices were not random. Such a comparison does not seem reasonable for commodity prices, however, because (1) contracts are generally only for a duration of about one year as opposed to several years for securities on the stock exchange, so no long-run trends due to inflation exist; (2) for every long position on the commodities market, there is a short position, a situation not true for the stock market; (3) there is no *a priori* reason on the commodities market to initially buy-and-hold, as opposed to sell-and-hold. Thus, the gross returns from the filters will be compared directly with the spectral results.[14]

Table 3 provides a comparison for each of the 30 contracts. As would be expected, those contracts judged not random generally generate the largest profits. Nevertheless, there were also some losses among these six. For the eleven almost random contracts the results seem to vary with both large gains and losses. However, for the thirteen random contracts gross profits on the average tend to be positive for all filter sizes, and the profits seem larger than might be expected. Some contracts, such as April 1965, June 1965, December 1965, and February 1969, generated gross profits for all six filters, while February 1967 generated some very large profits for certain filters. The October 1967, December 1967, and February 1968 contracts gave very mixed results.

These results seem to indicate that speculative profits may tend to be larger than might be expected given the spectral analysis results. Spectral analysis had already rejected, or created some doubt, about the random walk hypothesis for seventeen of the contracts, and the filter rules give additional evidence of price trends existing within the data by generating substantial profits. On the other hand, spectral analysis indicated beyond a doubt that the prices for thirteen of the contracts fluctuated randomly, but the filter rules continue to show positive gross (and often net) profits for many of these contracts. Thus, one would have to question that acceptance of the random walk hypothesis using statistical tools implies that no profits can be generated from the data by chart or mathematical devices [15]. Although we would have to concur in part, but not in general, with Cargill and Rausser [3] that "a simple stochastic process appears consistent with commodity markets price behavior," we would have to dispute their subsequent conclusion that in such instances there is no way of generating a profit by extrapolating from past changes in the prices.

This conflict is partly resolved by the fact that spectral analysis looks at time periods of fixed length (one day, in this study), while the filter rules allow the time period to vary, picking up nonlinear dependency. That is, short-run trends in the data may exist that spectral analysis cannot detect, but filter rules

14. It can be argued that profits after commission are not relevant to the random walk issue [5, p. 351], so using gross returns seems the most appropriate. However, the basic conclusions in this study would not be altered if net returns were used.

TABLE 3
COMPARISON OF SPECTRAL AND FILTER RULE RESULTS

Contract		Spectral Results	Filter Rule Results[d] 1%	2%	3%	4%	5%	10%
April	1965	R[a]	+ .90	+ 5.17	+ 3.52	+ 5.00	+ 4.32	+ 1.62
June	1965	R	+ 4.52	+10.32	+ 5.82	+ 3.07	+ 3.32	+ 6.50
August	1965	R	+ 8.35	+ 5.92	+ 3.70	− 5.45	+ 2.92	+ 3.27
October	1965	R	+ 1.20	+ 7.80	+ 1.57	− .02	− 3.42	+ 1.50
December	1965	R	+ 3.62	+ 3.27	+ 7.15	+ 2.52	+ .75	+ .95
February	1966	A.R.[b]	− 2.17	+ 3.00	+ 4.27	− .62	− 3.12	+ 2.62
April	1966	R	− 1.20	− .87	+ 3.75	+ 1.25	+ 4.62	+ 1.60
June	1966	A.R.	+ 5.55	+ 2.82	+ 7.22	+10.20	+ 7.32	− 4.82
August	1966	R	− 2.85	− 6.07	+ 2.87	+ 2.25	− .35	− .87
October	1966	A.R.	+ .62	+ 2.52	+ 7.27	+ 4.52	+ .87	+ 2.37
December	1966	A.R.	+13.97	+ 7.47	+ 6.60	− .37	+ 4.50	+ 5.42
February	1967	R	+11.25	+ 5.55	+ 4.52	+ 3.30	− 6.00	+ 2.17
April	1967	A.R.	+ 5.92	− 1.27	+ 3.57	+ 3.27	+ .62	+ 3.65
June	1967	A.R.	+ 9.97	+ 3.00	− 2.70	− .70	− 7.02	+ 1.20
August	1967	A.R.	+ 1.07	+ 2.55	+ 3.52	− 2.82	− 7.42	− 2.07
October	1967	R	+ 3.47	+ 1.57	+ 2.37	− 3.07	−10.37	− 1.50
December	1967	R	+ 3.42	+ 9.87	+ 2.27	− 4.40	−10.00	+ 1.67
February	1968	R	+ .02	+ 5.80	+ 4.55	− 1.40	− 5.90	− 2.97
April	1968	A.R.	+ 8.52	+ 4.47	+ 6.25	+ 4.62	+ 7.75	− 1.62
June	1968	A.R.	+ 1.55	+ 5.62	− .80	+ 1.67	+ 5.67	− 3.90
August	1968	A.R.	− 3.25	+ 1.00	+ 4.15	+ 5.15	+ 3.92	− 4.95
October	1968	R	+ 4.10	+ 2.97	− .07	− 1.95	+ 2.12	.00
December	1968	A.R.	+ 5.27	+ 8.62	+ 5.52	+ 2.10	+ 9.07	+ 2.85
February	1969	R	+ 7.07	+ 4.15	+ 5.07	+ 4.70	+ 2.75	+ 3.00
April	1969	N.R.[c]	+ 4.60	+ 7.70	+ 8.62	+ 1.32	+ 3.57	+ 7.40
June	1969	N.R.	+14.22	+ 4.97	+16.80	+16.05	+14.30	+17.17
August	1969	N.R.	+14.62	+12.65	+21.00	+12.57	+ 6.02	+ .45
October	1969	N.R.	+20.70	+ 6.65	+ 4.70	+ 1.15	+ .37	− 3.60
December	1969	N.R.	+14.65	+10.42	+ 5.60	− 2.65	−10.00	− 9.95
February	1970	N.R.	+14.15	+ 2.75	− 3.60	−11.02	−12.50	.00

[a] Random.
[b] Almost, or nearly, random.
[c] Significant variation from random walk model and judged not random.
[d] Gross returns in hundreds of dollars.

can. The lack of concurrence between the conclusions drawn from results generated by sophisticated statistical tools and mechanical filter rules in testing the random walk hypothesis on identical data demonstrates that one should be suspicious of the conclusions drawn by any researcher who used only one methodological approach and did not compare the results with an alternative approach.

For the average speculator interested in commodity futures trading, these results indicate that if he had used the 3 per cent rule on the 30 cattle contracts, substantial profits could have been generated, even after commissions. This occurs whether the price series is random or not. However, no probabilistic statement can be given as to the chances of that or any filter rule continuing to be profitable.

V. Conclusions

It has been the purpose of this paper to investigate the short-run fluctuations of the live beef cattle futures market with respect to the random walk hypothesis. Spectral analysis gave mixed results, indicating that a simple stochastic process appeared consistent with the price behavior of some of the contracts but not with others.

The same data were subjected to mechanical trading rules for further testing and comparison. These results cast serious doubt that cattle futures prices behave randomly. More importantly, the results allow one to seriously question the validity of drawing the conclusion that profitable trading is impossible after accepting the random walk hypothesis as the result of sophisticated statistical tools. Gross profits from trading rules seemed larger than might be expected where the spectral results indicated that prices behave in a random walk fashion. Prices which appear to be random as a result of statistical tests may still generate profits for the investor who relies on non-linear dependency through the use of filter rules, regardless of whether he pays commissions or not.

Thus, the conclusions drawn by any investigator who uses only one of the two basic approaches must be looked at with suspicion. This may call for using or developing alternative statistical tools to test the random walk hypothesis, especially tools that may detect the same price movements as do the mechanical filters. Also, further work is needed in investigating the accepted concept that anticipatory prices are random. Such an investigation should be made on the prices of those futures commodities where there is no inventory, such as with live cattle and hogs, being allocated over a period of time.

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